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Target/Background Polarization Profiles Using a COTS Digital Camera

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ABSTRACT

This paper develops the methodology to obtain polarization profiles of natural, day light scenes using a COTS (commercial-off-the-shelf) digital camera. It calculates the Stokes parameters for each scene pixel along with the degree of polarization, azimuth angle, sign and ellipticity for each of the color components. The noise characteristics of the digital camera are analyzed along with a detailed calibration of the imagery for target detection applications. A number of MatLab software programs were written including image analysis algorithms, graphics routines, polarization calculations and RGB value conversion to optical density. A pseudo color scheme was developed for encoding the various polarization parameters and comparing theoretical results with empirical data. Example imagery is presented of the generic methodology.

THE STOKES PARAMETERS

To determine the polarization properties of light, three independent quantities are necessary. For example, the state of polarization could be characterized from the amplitudes of the x and y components of the electric vector E_{0x} and E_{0y} and the phase difference δ. G. G. Stokes introduced a very convenient set of parameters in 1852 that are all of the same physical dimensions. These parameters may be determined for any given light wave from simple experiments. The Stokes parameters associated with a plane monochromatic light wave are defined as

$$
\begin{aligned}
S_0 &= E_{0X}^2 + E_{0Y}^2 \\
S_1 &= E_{0X}^2 - E_{0Y}^2 \\
S_2 &= 2\, E_{0X}\, E_{0Y} \cos \delta \\
S_3 &= 2\, E_{0X}\, E_{0Y} \sin \delta
\end{aligned} \tag{1}
$$

Only three of the Stokes parameters are independent since

$$S_0^2 = S_1^2 + S_2^2 + S_3^2.$$

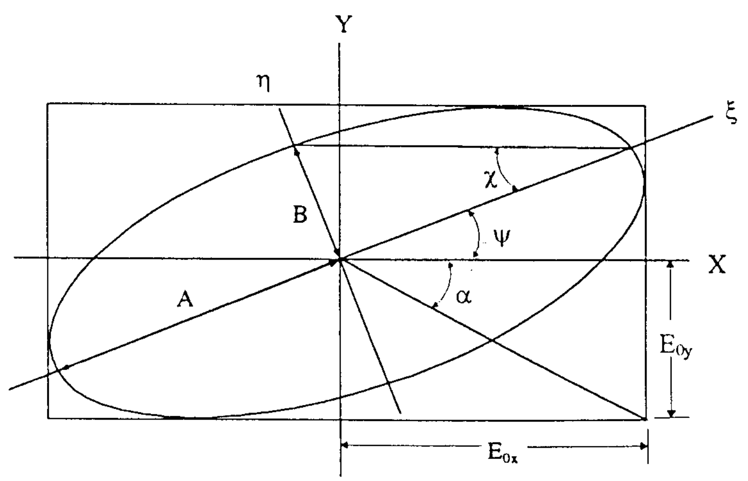

Figure 1. The polarization ellipse.

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Alternatively, the Stokes parameters can be expressed in terms of the polarization azimuth angle ψ and the ellipticity angle χ.

$$S_1 = S_0 \cos 2\chi \cos 2\psi$$
$$S_2 = S_0 \cos 2\chi \sin 2\psi$$
$$S_3 = S_0 \sin 2\chi \qquad (2)$$

For elliptically polarized light

$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 - 2\left(\frac{E_y E_x}{E_{0y} E_{0x}}\right)\cos\delta = \sin^2\delta \qquad (3)$$

MEASURING THE STOKES PARAMETERS

The x and y components of a plane light wave can be expressed as

$$E_x = E_{0x}\, e^{i\phi x} \quad \text{and} \quad E_y = E_{0y}\, e^{i\phi y} \qquad (4)$$

where E_{0x} and E_{0y} are the amplitudes of the x and y components of the incident light and ϕ_x and ϕ_y are the respective phases. If light is first transmitted through a linear retarder with its fast axis oriented at an angle Ω with respect to the x-axis and is then transmitted through a linear polarizer with its transmission axis oriented at an angle θ with respect to the x-axis, the intensity of the transmitted light can be expressed as

$$I(\Omega, \theta, \epsilon) =$$
$$E_{0x}^2 \{\cos^2\Omega \cos^2(\theta\text{-}\Omega) + \sin^2\Omega \sin^2(\theta\text{-}\Omega) + \sin 2\Omega \cos(\theta\text{-}\Omega) \sin(\theta\text{-}\Omega) \cos\varepsilon\} +$$
$$E_{0y}^2 \{\sin^2\Omega \cos^2(\theta\text{-}\Omega) + \cos^2\Omega \sin^2(\theta\text{-}\Omega) + \sin 2\Omega \cos(\theta\text{-}\Omega) \sin(\theta\text{-}\Omega) \cos\varepsilon\} +$$
$$E_{0x} E_{0y} \{\sin 2\Omega \cos\delta + \sin 2(\theta - \Omega)(\cos\varepsilon \cos\delta + \sin\varepsilon \sin\delta \cos 2\Omega)\} \qquad (5)$$

where $\delta = (\phi_x - \phi_y)$ is the phase difference between the x and y components of the incident light and ϵ is the phase difference produced by the retarder. $I(\Omega, \theta, \epsilon)$ denotes an intensity measurement corresponding to a particular set of values for Ω, θ and ϵ.

For a quartz quarter wave plate

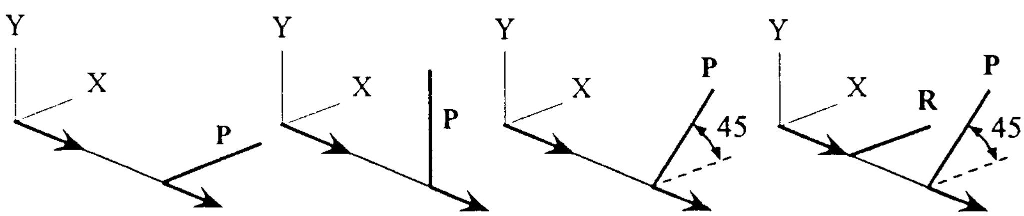
$$\varepsilon = \frac{\pi}{2}\left(\frac{\lambda_T - 50.876}{\lambda - 50.876}\right) \qquad (6)$$

where λ_T is the tuned wavelength and λ is the incident wavelength, both in nanometers.

The polarization state of the incident light can be determined experimentally from the following four measurements, as shown in Figure 2: $I_1 = I(0, 0, 0)$, $I_2 = I(0, 90, 0)$, $I_3 = I(0, 45, 0)$ and $I_4 = I(0, 45, \epsilon)$.

Using Eqs. (1) and (5)

$$S_0 = I(0, 0, 0) + I(0, 90, 0) = I_1 + I_2$$
$$S_1 = I(0, 0, 0) - I(0, 90, 0) = I_1 - I_2 ;$$
$$S_2 = 2\,I(0, 45, 0) - S_0 = 2\,I_3 - S_0$$
$$S_3 = \frac{2I_4 - S_0 - S_2 \cos\varepsilon}{\sin\varepsilon} \tag{7}$$



$I_1 = I(0, 0, 0)$ $I_2 = I(0, 90, 0)$ $I_3 = I(0, 45, 0)$ $I_4 = I(0, 45, \epsilon)$

Figure 2. Determining the Stokes parameters. **P** is the transmission axes of a linear polarizer and **R** is the fast axes of a linear retarder.

Using Eq. (2), the ellipticity χ and the azimuth angle ψ of the polarization ellipse can be determined from

$$\sin\ 2\chi = \frac{S_3}{S_0} \qquad \text{and} \qquad \tan\ 2\psi = \frac{S_2}{S_1} \tag{8}$$

For partially polarized light the degree of polarization **P** is given by

$$P = \frac{I_{polarized}}{I_{Total}} = \frac{\sqrt{\left(S_1^{\,2} + S_2^{\,2} + S_3^{\,2}\right)}}{S_0} \tag{9}$$

PRODUCING KNOWN POLARIZATION FORMS

The four Stokes parameters associated with a beam of light can be expressed as the elements of a 4 X 1 matrix, as given in Eq. (11). When light is incident on an optical device the Stokes parameters associated with the emerging beam are linear functions of the four Stokes parameters of the input beam. Equation (12) gives the linear transformation. An asterisk "*" is used on the Stokes parameters referring to the emerging beam.

$$\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \quad (10)$$

$$\begin{aligned} S_0^* &= M_{11}\,S_0 + M_{12}\,S_1 + M_{13}\,S_2 + M_{14}\,S_3 \\ S_1^* &= M_{21}\,S_0 + M_{22}\,S_1 + M_{23}\,S_2 + M_{24}\,S_3 \\ S_2^* &= M_{31}\,S_0 + M_{32}\,S_1 + M_{33}\,S_2 + M_{34}\,S_3 \\ S_3^* &= M_{41}\,S_0 + M_{42}\,S_1 + M_{43}\,S_2 + M_{44}\,S_3 \end{aligned} \quad (11)$$

Using matrix formulation, Eq. (11) can be expressed as

$$\begin{bmatrix} S_0 * \\ S_1 * \\ S_2 * \\ S_3 * \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \quad (12)$$

or equivalently as

$$\mathbf{S^*} = \mathbf{M}\,\mathbf{S} \quad (13)$$

where **S** is the Stokes column of the incident light, **S*** is the Stokes column of the light leaving the device and **M** is a 4 X 4 matrix which is characteristic of the device and its orientation and which is called, the Mueller matrix of the device.

Figure 3 illustrates how known polarization forms of different ellipticity can be produced using a linear polarizer and a retarder. It is an essential step in the process of testing the results obtained from the digital camera. The resultant Stokes parameters for any given polarization form can be obtained from Eq. (13) by using the appropriate Mueller matrix **M**.

The Mueller matrix $\mathbf{M_P}$ for an ideal linear polarizer with transmission axis at angle α from the x-direction is

$$M_P = \frac{1}{2}\begin{bmatrix} 1 & \cos(2\alpha) & \sin(2\alpha) & 0 \\ \cos(2\alpha) & \cos^2(2\alpha) & \cos(2\alpha)\ \sin(2\alpha) & 0 \\ \sin(2\alpha) & \cos(2\alpha)\ \sin(2\alpha) & \sin^2(2\alpha) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (14)$$

The Mueller matrix $\mathbf{M_R}$ for an ideal linear retarder with retardation ε and fast axis at angle β from the x-direction is

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2(2\beta)+\sin^2(2\beta)\ \cos(\varepsilon) & \cos(2\beta)\ \sin(2\beta)\ (1-\cos\ \varepsilon) & -\sin(2\beta)\ \sin(\varepsilon) \\ 0 & \cos(2\beta)\ \sin(2\beta)\ (1-\cos\ \varepsilon) & \sin^2(2\beta)+\cos^2(2\beta)\ \cos(\varepsilon) & \cos(2\beta)\ \sin(\varepsilon) \\ 0 & \sin(2\beta)\ \sin(\varepsilon) & -\cos(2\beta)\ \sin(\varepsilon) & \cos(\varepsilon) \end{bmatrix} \quad (15)$$

The Stokes vector for unpolarized light is given by Eq. (16). The resultant Stokes vector **S*** for unpolarized light transmitted through an ideal linear polarizer with transmission axis at angle α from x-direction is found from **S*** = $\mathbf{M_P}$ **S**. The result is given by Eq. (17). If light is transmitted through an ideal linear retarder with retardation ε and fast axis at angle β from the x-direction, the resultant Stokes vector is found from the product of Eq. (15) and Eq. (17).

$$S = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (16) \qquad\qquad S^* = \frac{1}{2}\begin{bmatrix} 1 \\ \cos(2\alpha) \\ \sin(2\alpha) \\ 0 \end{bmatrix} \qquad (17)$$

The resultant normalized Stokes parameters for the beam emerging from the retarder are given by

$$\begin{aligned} S_0 &= 1 \\ S_1 &= \cos 2(\alpha-\beta) \cos 2\beta - \sin 2(\alpha-\beta) \sin 2\beta \cos \varepsilon \\ S_2 &= \cos 2(\alpha-\beta) \sin 2\beta + \sin 2(\alpha-\beta) \cos 2\beta \cos \varepsilon \\ S_3 &= -\sin 2(\alpha-\beta) \sin \varepsilon \end{aligned} \qquad (18)$$

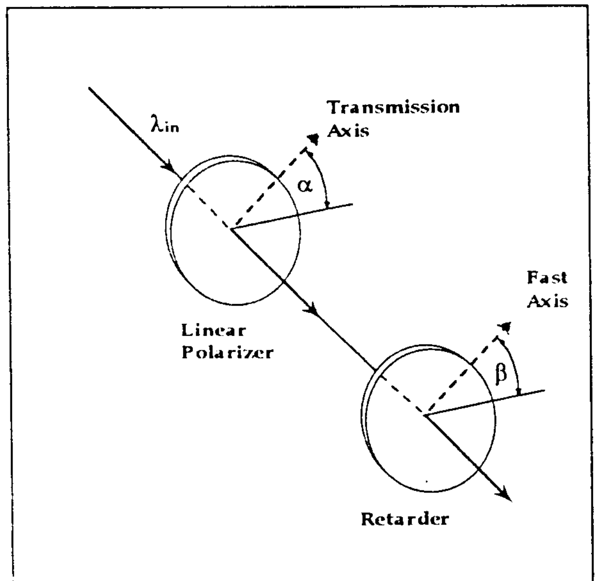


Figure 3. Setup for producing known. polarization forms.

DIGITAL CAMERA NOISE

Digital cameras are solid-state light detectors, CCDs, which stand for "charged-coupled device". They detect light at thousands of photosites which are ten times more sensitive than the fastest photographic films. At each photosite, the incident light frees electrons and charged-coupling moves the electrons to an amplifier which produces a digitized output. Unfortunately, the CCDs are not perfect. Thermal signals, or "dark current", increase with time and the zero point is biased above true zero. Noise in many different forms is added to the signal at each photosite. The number of photons that reach the CCD during each exposure varies randomly. A signal to noise ratio of 10 is a reasonably solid detection. Unfortunately, the light-sensing photosites generate a signal whether light is incident on them or not. Thermal noise increases with the temperature of the CCD; the warmer the CCD, the greater the non-light signal. Analysis of a picture taken with a lens cap over the camera lens, a *dark image frame*, will reveal the extent of thermal noise.

Readout noise results from random statistical variations in the amplifier and other electronics of the CCD camera. It occurs because the amplifier cannot determine exactly how many electrons have come from each photosite on the chip. Quantization noise results from having digital data. The amplifier counts the number of electrons from each "well" of electric charge and produces a

proportional output voltage that is sent to an anolog-to-digital converter. An 8-bit camera divides the signal into 256 steps, a 12-bit camera divides the signal into 4,096 steps and a 16-bit camera divides the signal into 65,536 steps. The photosites of most CCDs today are uniform to approximately 10% over the surface of the chip. A *flat-field frame*, an image of a uniformly-illuminated surface, will produce a map of the CCD's sensitivity to light. Figure 6 shows the results of a MatLab analysis of a flat-field frame from an Epson 850Z digital camera acquired from a clear, blue, uniform sky. In a perfect world, there would be a single value for each color channel.

DESIRABLE CAMERA FEATURES

A most desirable feature of any digital camera to be used to acquire polarization profiles is a dark image frame with no non-zero values for any of its channels. Figure 4 shows the results of a MATLAB analysis of a dark image frame acquired from a SBIG ST-7 digital camera. Figure 5 shows the results of a MATLAB red channel analysis of a dark image frame acquired from a Sony MVC-FD91 Mavica digital camera. Evidently, the characteristics of a dark image frame are different for each kind of digital camera. Surprisingly, an analysis of the dark image frame of an Epson 850Z digital camera showed absolutely no non-zero values for any of the channels! Obviously, for this camera, thermal noise is subtracted out prior to the determination of the RGB values.

In addition to the minimization of all forms of noise, to acquire polarization profiles, the digital camera must have the manual ability to set the aperture and the shutter speed. The white balance must be able to be fixed and spot metering should be available. The camera should have sensitivity selections and the option for timed pictures is very desirable for registration control. The ability of having a monochrome/color selection allows for the option of inserting ones own filters in front of the lens.

CALIBRATION – RGB TO OPTICAL DENSITY

An 8-bit camera must convert the light intensity at each photosite into an RGB value, an integer between 0 and 255. To determine the mathematical nature of this conversion, a Macbeth color checker was photographed with different neutral density filters in front of the lens of the digital camera. Neutral density filters have known optical densities which reduce the transmission through them according to

$$I_T = I_0\ 10^{-D} \tag{19}$$

where D = Optical Density, I_0 = Incident intensity and I_T = Transmitted intensity.

The achromatic objects on the Macbeth chart also correspond to known optical densities. Figure 7 shows the result of a MatLab analysis of the green channel values corresponding to the achromatic objects on the Macbeth color checker when recorded through many different neutral density filters.

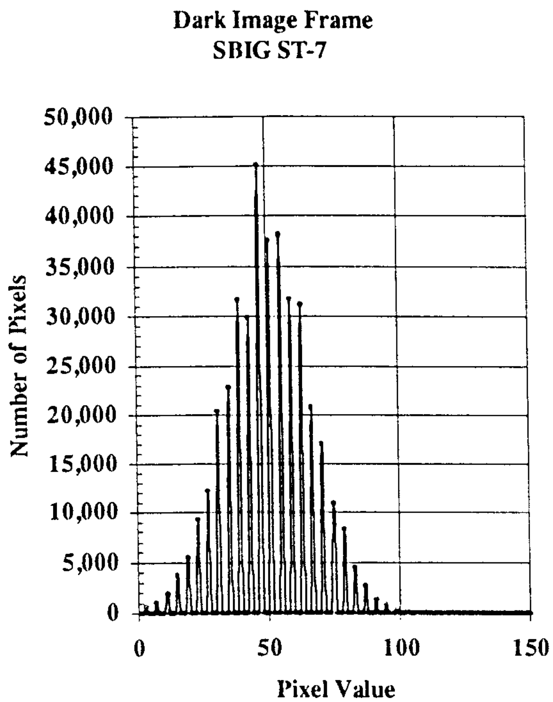


Figure 4. Non-Zero Values of a Dark Image Frame from a SBIG ST-7.

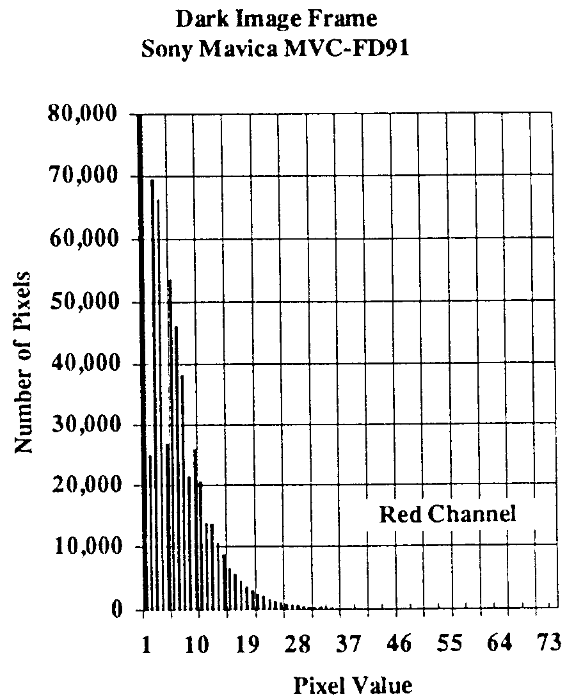


Figure 5. Non-Zero Values of a Dark Image Frame from a Sony Mavica MVC FD91.

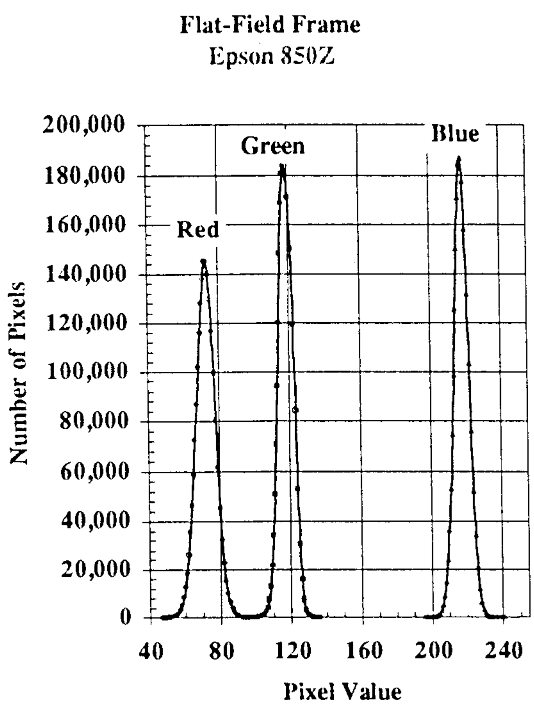


Figure 6. Flat-Field Frame of Epson 850Z.

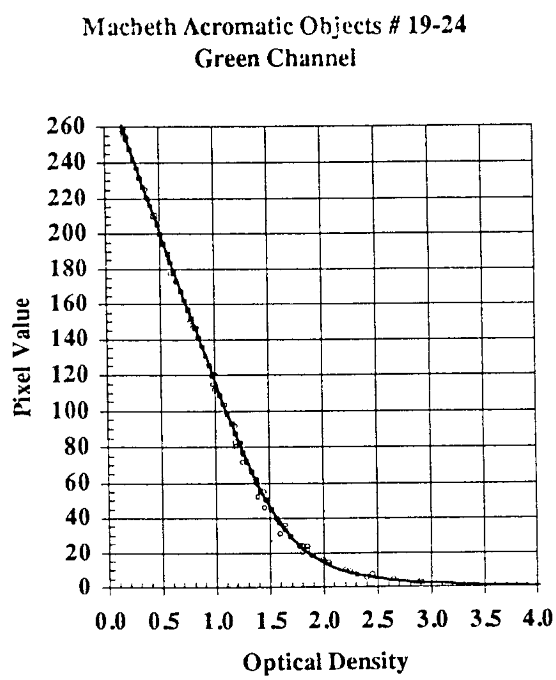


Figure 7. Conversion from RGB to Optical Density for Green Channel of Epson 850Z.

The best trendline through the points was determined to be a hyperbola of the form

$$A x^2 + B xy + C y^2 + D x + E y + F = 0 \qquad (20)$$

where x and y correspond to the optical densities and RGB values respectfully. The coefficients in Eq. (20) are different for each color channel of the digital camera. Using the fixed white balance option of the Epson 850Z, the coefficients obtained for the green channel are: A = C = 0.19926, B = 35.67852, D = 127.49000, E = -55.65087 and F = -520.1110.

RESULTS

A simple test was performed to determine the validity of Eq. (20). It was based on the law of Malus which gives the intensity of the light transmitted through crossed polarizers. It states that

$$I = I_0 \cos^2 \theta \qquad (21)$$

where I is the transmitted intensity, I_0 is the incident intensity and θ is the angle between the transmission axes of two ideal linear polarizers. Figure 8 shows the results of the study. The circular markers where obtained from ten different digital images of two crossed linear polarizers. Each image corresponded to a different angle θ. Using MatLab, the RGB values were extracted from the same pixel location in each image. The solid line in Figure 8 corresponds to the law of Malus.

Another important test of the accuracy of the Epson 850Z to determine polarization profiles involved recording images of the transmitted light through the setup shown in Figure 3. Sets of four images were acquired for many different linear and elliptically polarized forms using both incident quasi-monochromatic and white light. These images were acquired using the scheme shown in Figures 2-5 for determining the Stokes parameters. Using MatLab, the Stokes parameters were obtained from a 10 X 10 pixel area using Eq. (20) for each pixel in the scene. Equations (8) and (9) were used to derive individual images of the degree of polarization P, the polarization azimuth angle ψ and the ellipticity angle χ. The RGB colorization scheme shown in Figure 9 was used to represent different polarization angles. Figures 10-11, generated using Eqs. (1) and (3), compare the predicted polarization forms to those actually obtained using an Epson 850Z digital camera. The results shown in Figures 10 were obtained from quasi-monochromatic light of wavelength 546 nm transmitted through the setup shown in Figure 3. The results shown in Figures 11 were obtained for white light transmitted through the setup shown in Figure 3. The tuned wavelength of the retarder was 546 nm. A wavelength of 633 nm was used for the red channel and 546 nm was used for the green channel of the Epson 850Z.

Figure 12 gives the polarization profiles of a vehicle in daylight. The degree of polarization P, the polarization azimuth angle ψ and the ellipticity χ are shown for two times during daylight, 7:00 and 12:00. P = 1 for the white areas and P = 0 for the black areas in Figure 12. The polarization angles ψ and χ were colorized using the scheme shown in Figure 9.

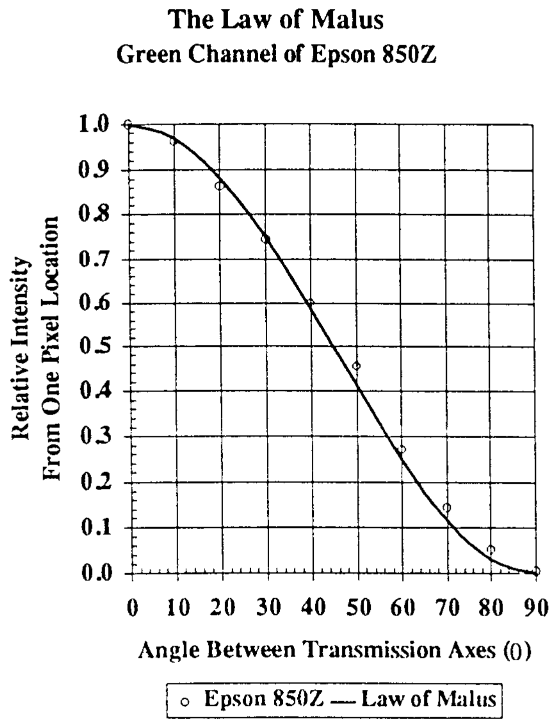


Figure 8. Testing the law of Malus.

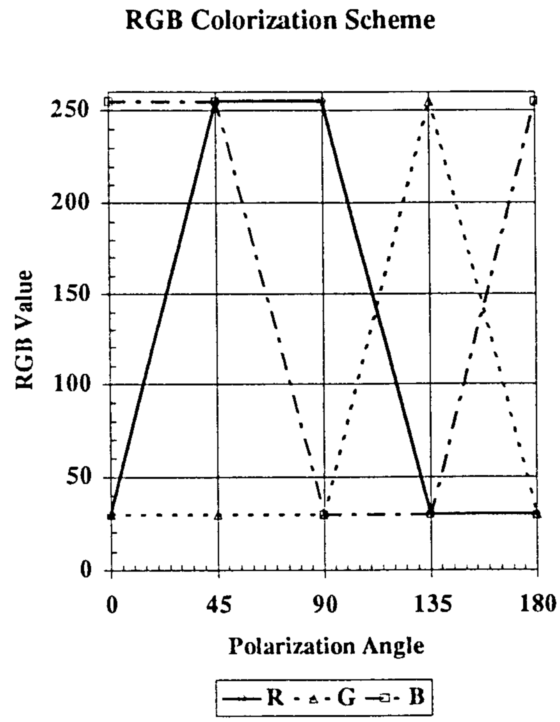


Figure 9. RGB colorization scheme for polarization angles.

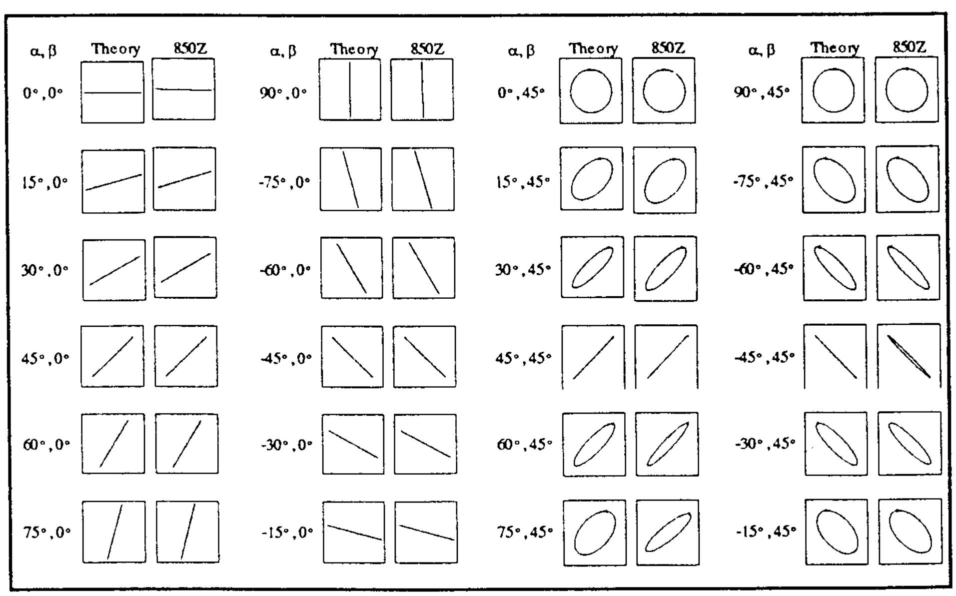


Figure 10. Comparison of known polarization profiles produced with the setup shown in Figure 3 and those acquired from the Green Channel of an Epson 850Z Digital Camera. The input wavelength was 546 nm and the tuned wavelength of the retarder was also 546 nm.

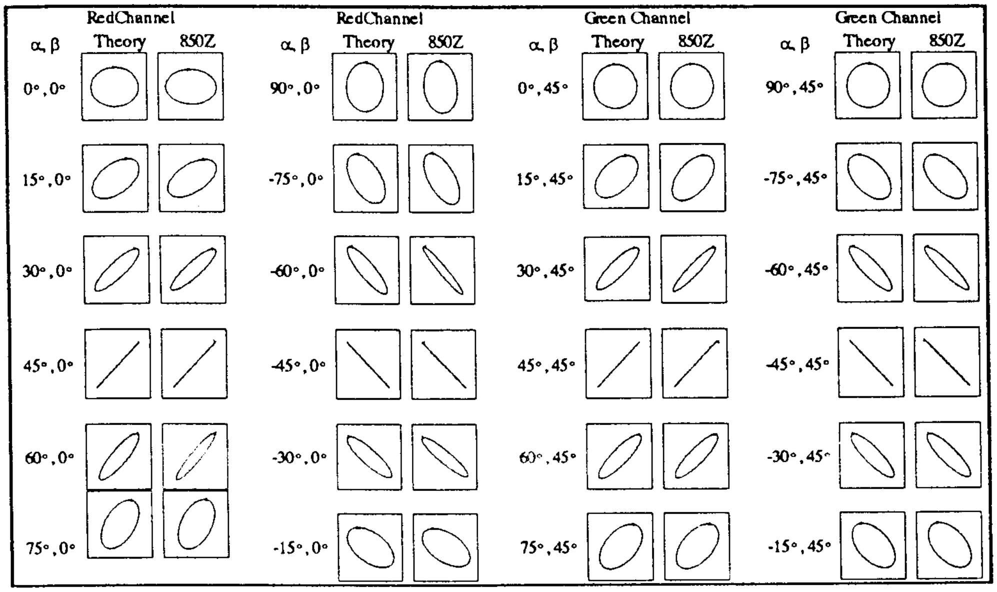


Figure 11. Comparison of known polarization profiles produced with the setup shown in Figure 3 and those acquired from the red and green channels of an Epson 850Z digital camera. The incident light was white light and the tuned wavelength of the retarder was 546 nm. A wavelength of 633 nm was used for the red channel and 546 nm was used for the green channel.

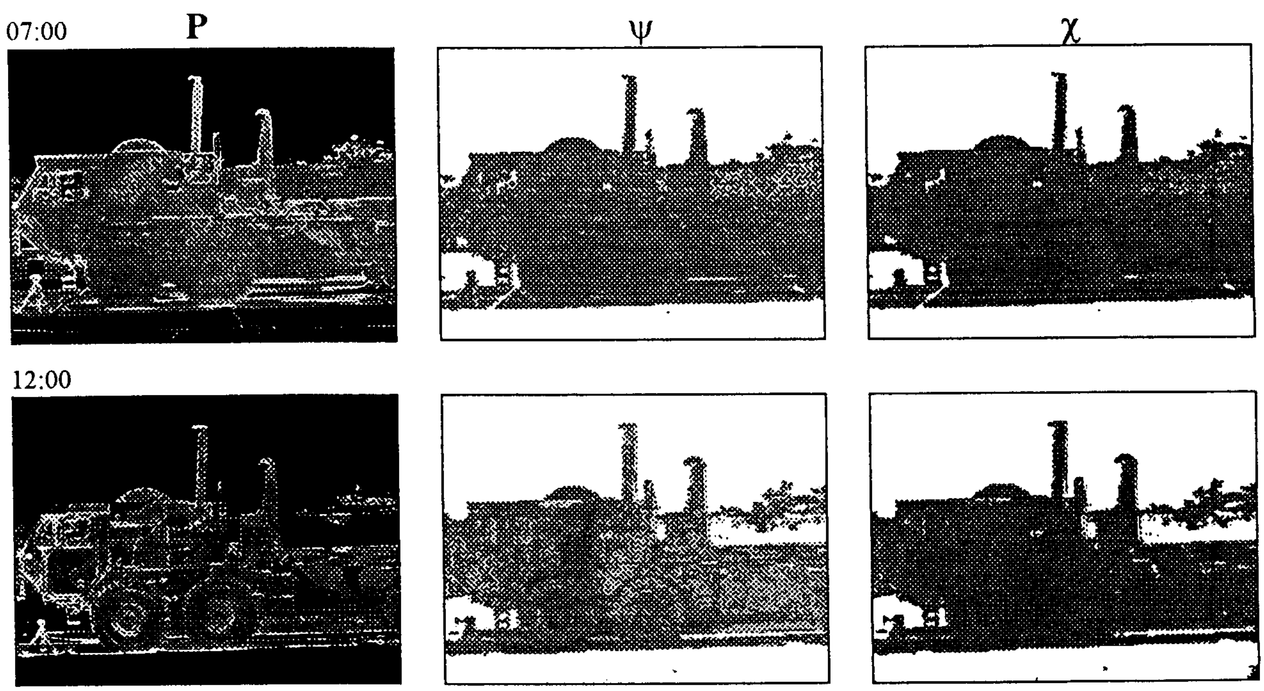


Figure 12. The polarization profiles of a vehicle in daylight. The degree of polarization P, the polarization azimuth angle ψ and the ellipticity χ are shown for the two times 7:00 and 12:00.

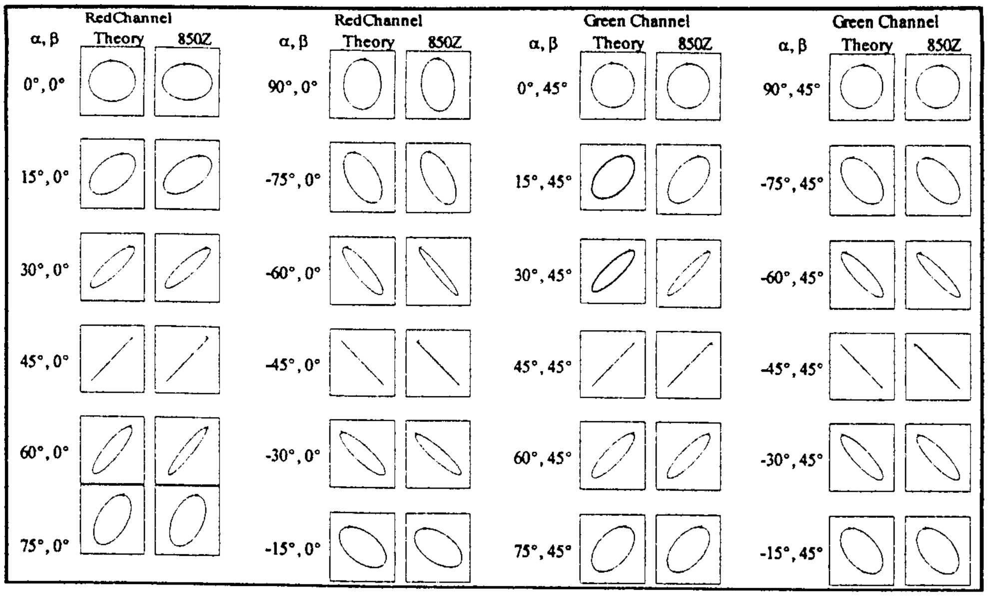


Figure 11. Comparison of known polarization profiles produced with the setup shown in Figure 3 and those acquired from the red and green channels of an Epson 850Z digital camera. The incident light was white light and the tuned wavelength of the retarder was 546 nm. A wavelength of 633 nm was used for the red channel and 546 nm was used for the green channel.

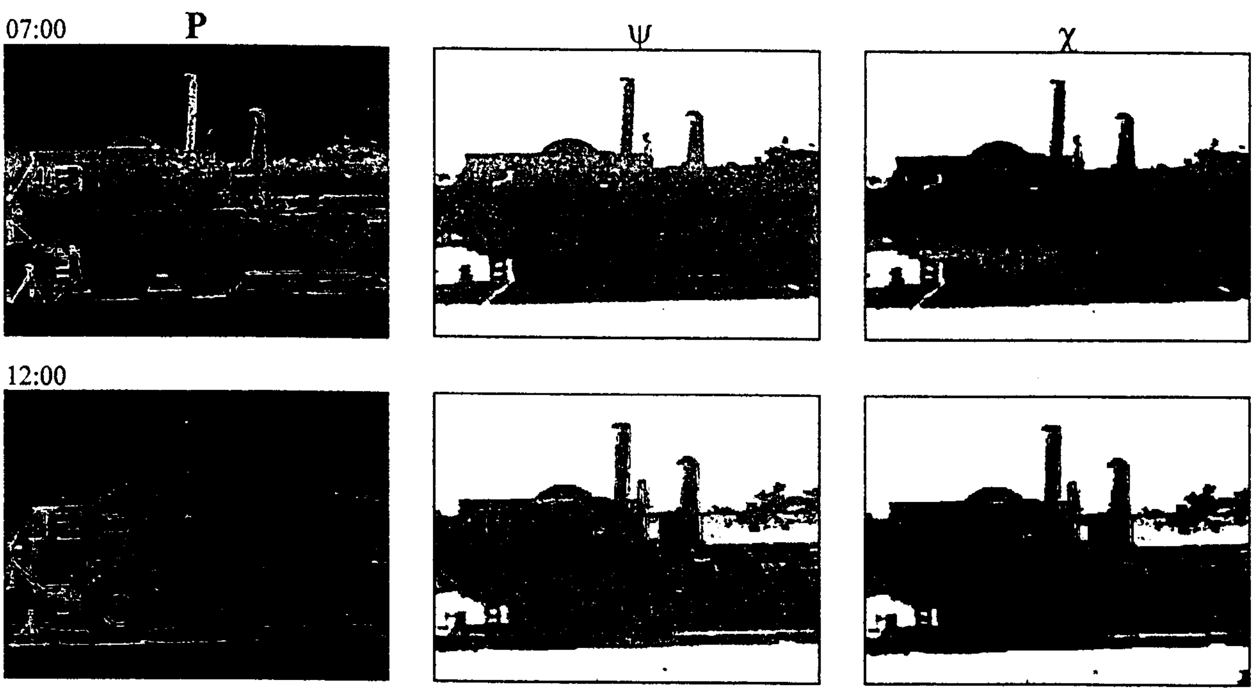
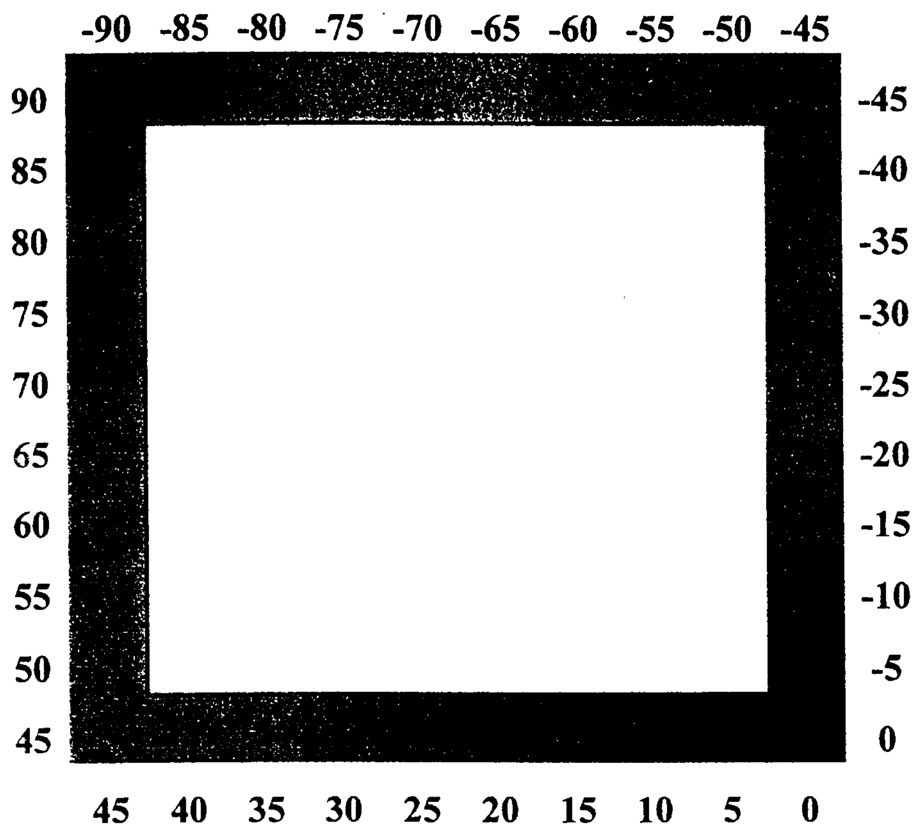


Figure 12. The polarization profiles of a vehicle in daylight. The degree of polarization P, the polarization azimuth angle ψ and the ellipticity χ are shown for the two times 7:00 and 12:00.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90 85 80 75 70 65 60 55 50 45
-45 -40 -35 -30 -25 -20 -15 -10 -5 0
45 40 35 30 25 20 15 10 5 0